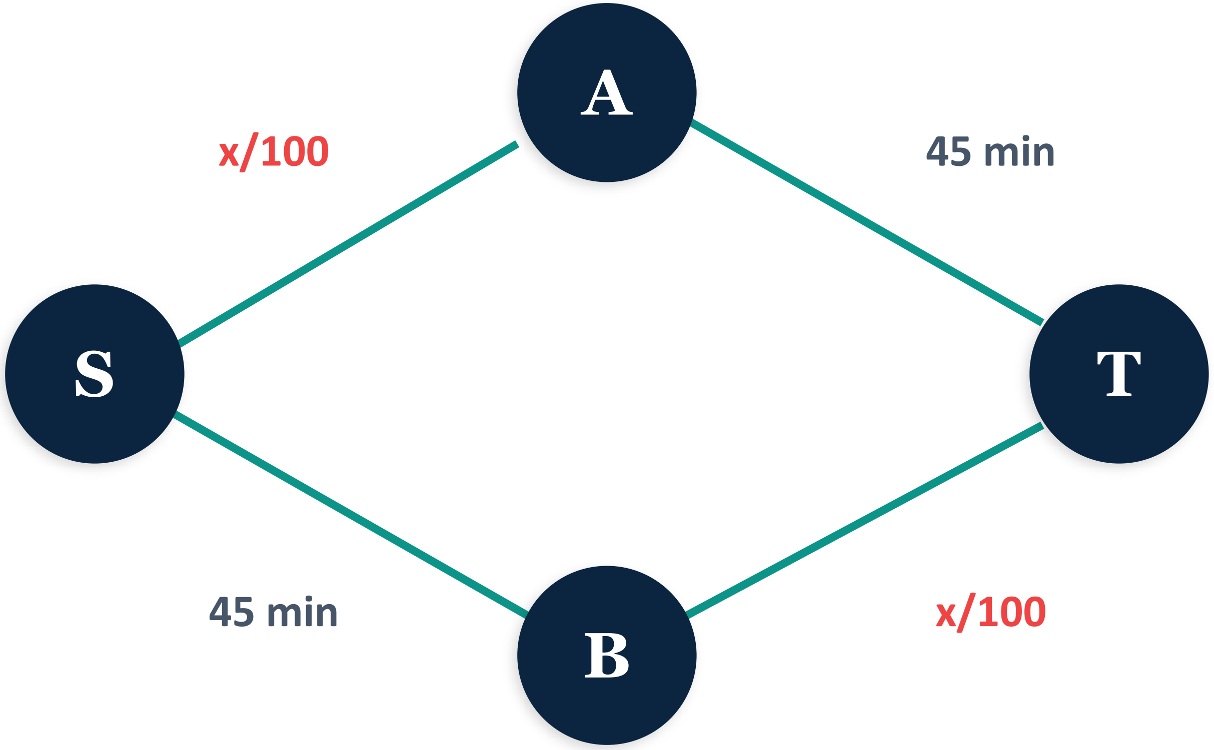
Braess's Paradox

Why teleporters can harm your commute

Congestion Games



Setup

4,000 drivers travel from S to T.

Travel time on some roads depends on how many drivers use that road

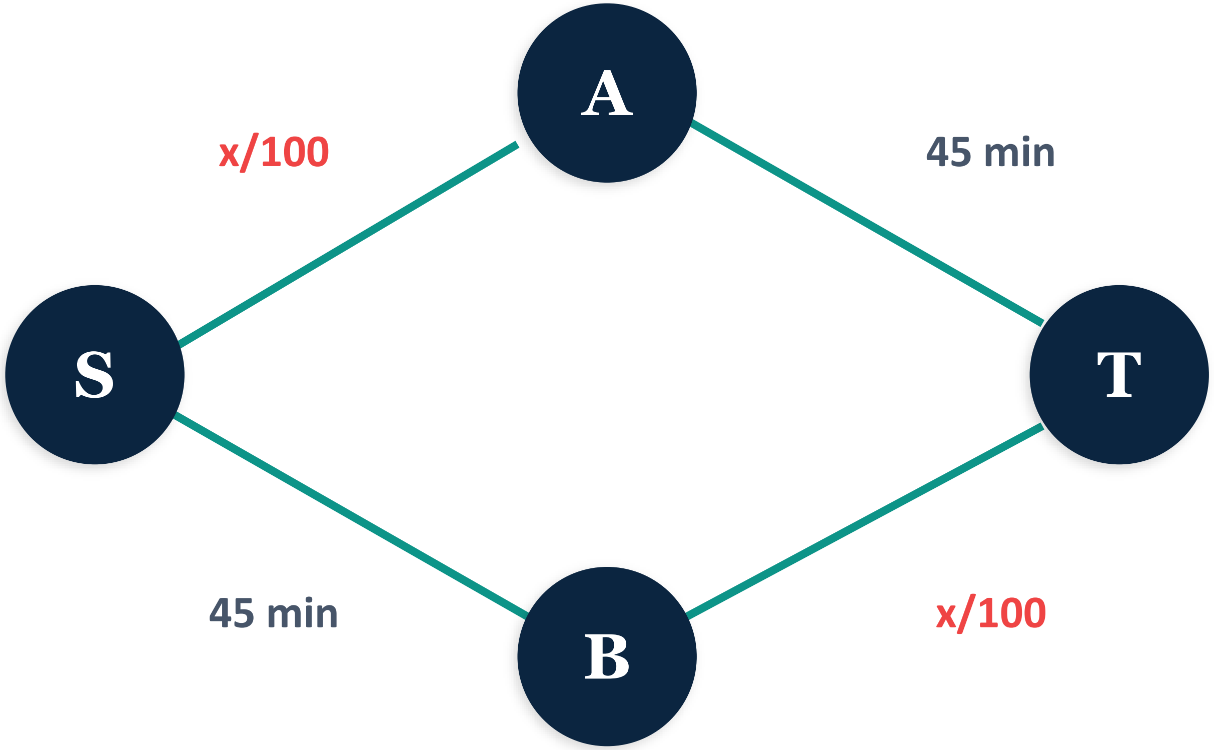
Some roads are large with a constant travel time.

Goal: allocate drivers to minimize average commute time.

Solution = 2,000 on each route.
Cost = 65 min per driver.

This solution is a **Nash Equilibrium:** no individual driver wants to switch

Price of Anarchy



PoA = 1

Selfish behavior causes no harm.

PoA > 1

Selfish agents degrade system performance.

Big Question:
What is the overall cost of self-interested behaviour?

For a particular traffic network, define two quantities:

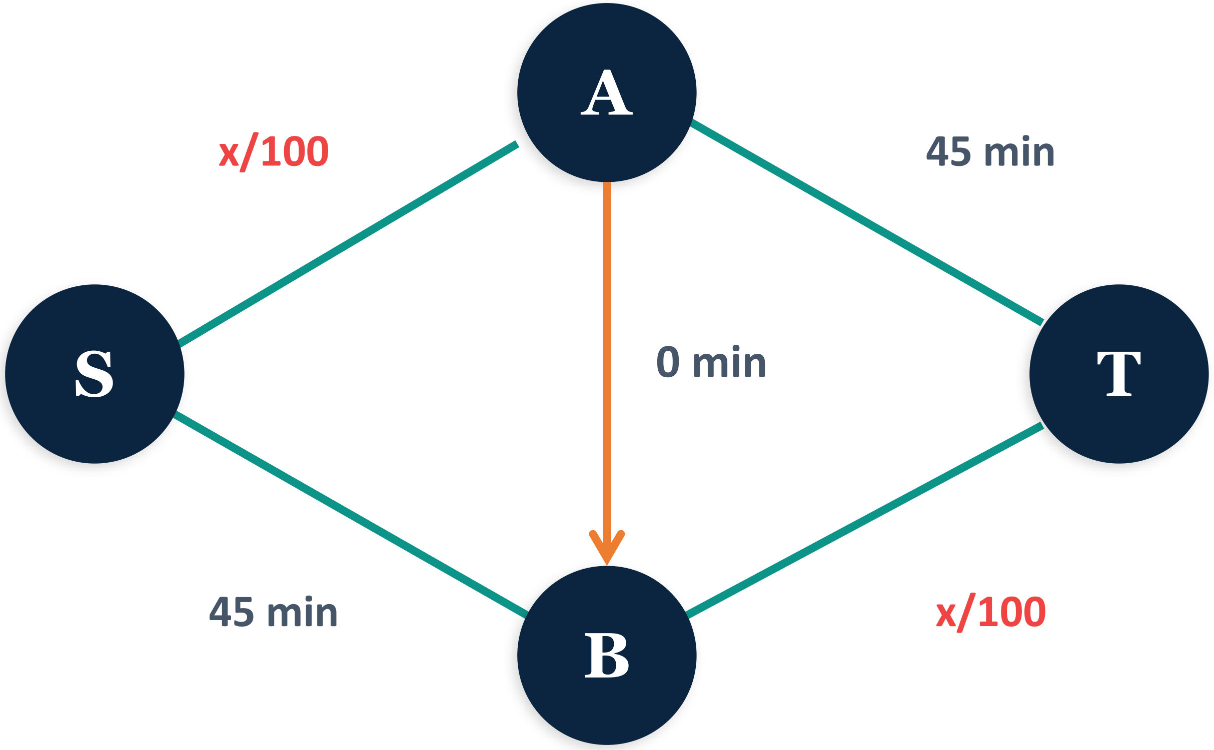
Selfish Routing: average commute time in a Nash Equilibrium

Optimal Routing: average commute time with central planner (AKA dictator)

Price of Anarchy:

$$\frac{\text{cost(selfish routing)}}{\text{cost(optimal routing)}}$$

Adding a Shortcut



What changes?

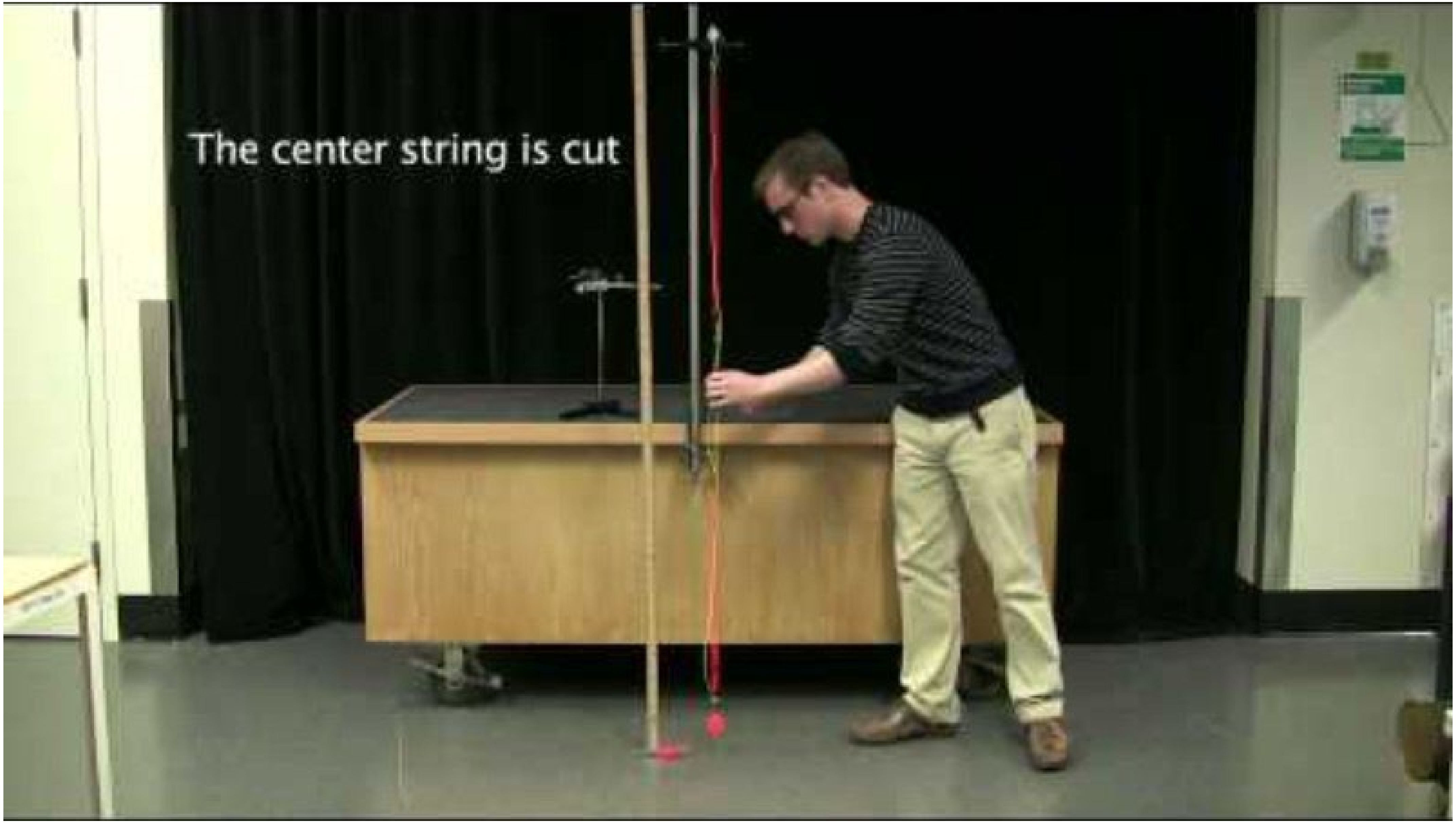
4,000 drivers travel from S to T.

Selfish routing?
Everyone uses the teleporter!
Commute time = 80 min per driver.

What is the optimal routing?
Solution: 500 people use the teleporter
Avg. commute time = 64.6875 min.

Price of Anarchy = 1.237

Under agentic dynamics**, building a teleporter harms everyone** ☹



Physical Analogy to Springs

Real-World Examples

Stuttgart, Germany

In 1969, a new road was built downtown. Traffic worsened. Closing the road improved flow.

Seoul, South Korea

In 2003, a six-lane highway was demolished and replaced with a park (Cheonggyecheon). Traffic congestion decreased citywide.

New York City, USA

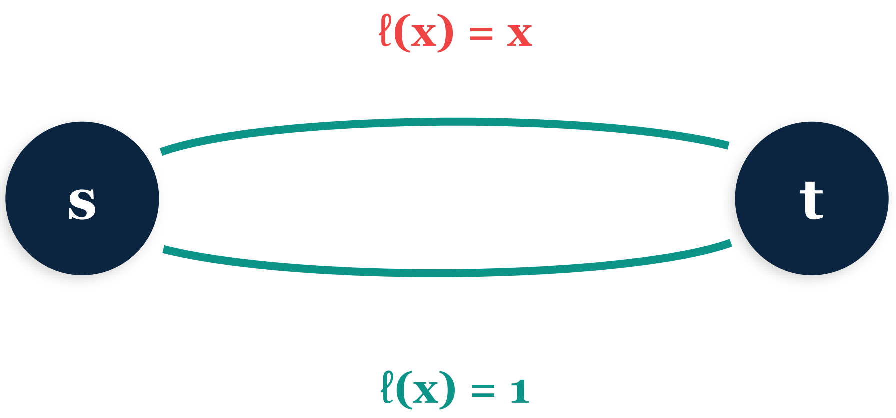
On Earth Day 1990, 42nd Street was closed. Predicted gridlock never materialized.

Power Grids & Internet

The paradox extends beyond roads: adding transmission lines or network links can degrade system performance under selfish usage.

https://en.wikipedia.org/wiki/Braess's_paradox

How bad can it be?



Social optimum: Split 50/50. Avg cost = 3/4.

Nash equilibrium: All flow on top edge. Cost = 1.

PoA = 4/3

Roughgarden & Tardos (2002):
In networks with **linear cost functions** ℓ(x) = ax + b, the Price of Anarchy is **always at most 4/3**.

Selfish routing (at equilibrium) is never more than 33% worse than the social optimum.

For polynomials of degree p, PoA grows as ≈ p / ln p. More severe, but still independent of n!

Thank you